

Indian Institute of Technology Kharagpur QUESTION-CUM-ANSWERSCRIPT										Stamp/Signature of the Invigilator			
MID-SEMESTER EXAMINATION							SEMESTER (AUTUMN-2022)						
Roll Number										Section		Name	
Subject Number		M	A	1	1	0	0	3		Subject Name	Advanced Calculus		
Department/Centre/School													
Important Instructions and Guidelines for Students													
1. You must occupy your seat as per the Examination Schedule/Sitting Plan. 2. Do not keep mobile phones (smart watch) or any similar electronic gadgets with you even in the switched off mode. 3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination. 4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter. 5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted. 6. Write on both sides of the answer-script and do not tear off any page. Report to the invigilator if the answer-script has torn or distorted page(s). 7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator. 8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall. 9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. In any case, you are not allowed to take away the answer-script with you. After the completion of the examination, do not leave your seat until invigilators collect all the answer-scripts. 10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as 'unfair means'. Don't adopt unfair means and also don't indulge in unseemly behavior. 11. Please see overleaf for more instructions <i>Violation of any of the above instructions may lead to severe punishment.</i>													
													Signature of the Student
To be Filled by the Examiner													
Question Number	1	2	3	4	5								Total
Marks Obtained													
Marks Obtained (in words)				Signature of the Examiner					Signature of the Scrutinizer				

Instructions and Guidelines to the Students

1. No queries will be entertained during the examination.
2. Examination is of two hours duration.
3. The question-cum-answer booklet has 24 pages and 5 questions with a total of 30 marks.
4. All the questions are compulsory.
5. **Answer each question in the space provided below to that question only. Otherwise it will not be evaluated.**
6. **Use supplementary sheet for rough work. Any answer written in the supplementary sheet will not be evaluated.**
7. **Notations:** $\mathbb{R}$ denotes the set of real numbers.

1. (a) Let $f(x) = a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \cdots + a_n \sin nx$, where $a_1, a_2, a_3, \dots, a_n \in \mathbb{R}$. Also, it is given that $|f'(x)| \leq |\cos x|$ for all $x \in \mathbb{R}$. Then using **Lagrange's Mean Value Theorem** (LMVT), prove that $|a_1 + 2a_2 + 3a_3 + \cdots + na_n| \leq 1$.
- (b) Write Maclaurin series for $\sqrt{1+x+x^2}$ up to degree 2 polynomial along with the corresponding Lagrange's form of remainder. Use this polynomial to estimate $\sqrt{1.75}$.

[3 + 3 Marks]

2. (a) Test the continuity of the following function at the origin $(0, 0)$.

$$f(x, y) = \begin{cases} \frac{y^2 - x}{y^4 + x^2} & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } (x, y) = (0, 0). \end{cases}$$

(b) For the function $f(x, y) = \begin{cases} \frac{x^3}{x^2 + y^2} & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } (x, y) = (0, 0), \end{cases}$

compute $\frac{\partial f(x, y)}{\partial x}$ for all $(x, y) \in \mathbb{R}^2$ and discuss the continuity of $\frac{\partial f(x, y)}{\partial x}$ at the origin $(0, 0)$.

[3 + 3 Marks]

3. (a) Test the differentiability of the following function at the origin

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^4 + y^2} & , \text{if } (x, y) \neq (0, 0). \\ 0 & , \text{if } (x, y) = (0, 0). \end{cases}$$

- (b) If $z = f(x, y)$, $x = e^{2u} + e^{-2v}$, $y = e^{-2u} + e^{2v}$, then show that $\frac{\partial f}{\partial u} - \frac{\partial f}{\partial v} = 2 \left(x \frac{\partial f}{\partial x} - y \frac{\partial f}{\partial y} \right)$.

[3+3 Marks]

4. (a) Let $u^3 = \tan^{-1} \left(\frac{x^n + y^n + z^n}{\sqrt{x^4 + y^4 + z^4}} \right) + 3$. Using Euler's theorem, find the value of n such that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = \frac{\sin(2u^3 - 6)}{u^2}.$$

- (b) Compute $\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) (0, 0)$ for the following function

$$f(x, y) = \begin{cases} \frac{xy^2(x - 2y)}{x^2 + y^2} & , \text{ if } (x, y) \neq (0, 0), \\ 0 & , \text{ if } (x, y) = (0, 0). \end{cases}$$

[3 + 3 Marks]

5. (a) Find and classify the points where a saddle, local maximum or local minimum occurs for the function $f(x, y) = xye^{-2(x^2+y^2)}$.
- (b) Let $P_2(x, y) = ax^2 + bxy + \frac{y^2}{2} + x - y + 1$ be the Taylor polynomial of degree 2 about the point $(0, 0)$ for the function $f(x, y) = e^{x-y}(4x^2 - 3xy + 1)$, where a, b are real numbers. Then find the values of a, b .

[3 + 3 Marks]