

Indian Institute of Technology Kharagpur																			
QUESTION-CUM-ANSWERSCRIPT										Stamp/Signature of the Invigilator									
END-SEMESTER EXAMINATION										SEMESTER (Autumn-2023)									
Roll Number										Section			Name						
Subject Number		M	A	1	1	0	0	3		Subject Name		Advanced Calculus							
Department/Centre/School																			
Important Instructions and Guidelines for Students																			
1. You must occupy your seat as per the Examination Schedule/Sitting Plan. 2. Do not keep mobile phones (smart watch) or any similar electronic gadgets with you even in the switched off mode. 3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination. 4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter. 5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted. 6. Write on both sides of the answer-script and do not tear off any page. Report to the invigilator if the answer-script has torn or distorted page(s). 7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator. 8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall. 9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. In any case, you are not allowed to take away the answer-script with you. After the completion of the examination, do not leave your seat until invigilators collect all the answer-scripts. 10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as 'unfair means'. Don't adopt unfair means and also don't indulge in unseemly behavior. 11. Please see overleaf for more instructions. <i>Violation of any of the above instructions may lead to severe punishment.</i>																			
Signature of the Student																			
To be Filled by the Examiner																			
Question Number		1	2	3	4	5				Total									
Marks Obtained																			
Marks Obtained (in words)										Signature of the Examiner					Signature of the Scrutinizer				

Instructions and Guidelines to the Students

1. No queries will be entertained during the examination.
2. Examination is of three hours duration.
3. The question-cum-answer booklet has 40 pages and 5 questions with a total of 50 marks.
4. All the questions are compulsory.
5. Answer each question in the space provided below to that question only. Otherwise it will not be evaluated.
6. Use last eight pages for rough work. Any answer written in the rough sheet will not be evaluated. No additional sheets will be provided for rough work.
7. **Notations:** $\mathbb{R}$ denotes the set of real numbers.

1. (a) Let $f : [a, b] \rightarrow \mathbb{R}$ be a differentiable function and $a \geq 0$. Using Cauchy's Mean Value Theorem, prove that there exist $c_1, c_2 \in (a, b)$ such that

$$\frac{f'(c_1)}{a+b} = \frac{f'(c_2)}{2c_2}.$$

- (b) Check differentiability of the function

$$f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{2x^2 + 3y^2}, & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0), \end{cases}$$

at the origin $(0, 0)$.

- (c) Determine the second degree Taylor's polynomial for the function

$$f(x, y) = xe^y + 1,$$

about the point $(1, 0)$.

[3 + 4 + 3 Marks]

2. (a) Find the general solution of $(D^2 - 2D + 2)^2(D^2 - 5D + 4)y = 0$. Here, $D = \frac{d}{dx}$
- (b) Find the general solution of the differential equation

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 16x^3 \sin^2 x, \quad x > 0$$

by evaluating the particular solution using the method of variation of parameters.

- (c) By using the D -operator method, find a particular solution y_p of the differential equation $(D^3 - 4D^2 + 13D)y = 78e^{2x} \cos(3x)$, where $D = \frac{d}{dx}$. **[2 + 5 + 3 Marks]**

3. (a) Using triple integral, find the volume of the solid enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.
- (b) Find the directional derivative for the function $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction $3\hat{i} - 4\hat{j}$.
- (c) Applying differentiation under integration sign rule, evaluate

$$\lim_{x \rightarrow 0} \left[\frac{d}{dx} \left(\int_{2x-1}^{3x+2} \left(\frac{t+6}{4t} \right)^t dt \right) \right].$$

[4 + 2 + 4 Marks]

4. (a) Using Beta function evaluate $\int_0^1 \sqrt[11]{1 - \sqrt[3]{x}} dx$.
- (b) Evaluate the following integral by changing the order of integration
- $$\int_{x=0}^{x=\frac{a}{\sqrt{2}}} \int_{y=x}^{y=\sqrt{a^2-x^2}} y^2 dy dx.$$
- (c) Test for convergence of the integral

$$\int_0^\infty f(x) dx, \quad \text{where } f(x) = \begin{cases} \frac{\sin\left(\frac{3x}{4}\right)}{x^{\frac{3}{4}}} & \text{if } x \geq \pi \\ \frac{1 - \cos(\sqrt{x})}{\sqrt{\pi - x}} & \text{if } 0 \leq x < \pi. \end{cases}$$

[3 + 3 + 4 Marks]

5. (a) Obtain the solution of the differential equation

$$[(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)] y = \frac{1}{x+1}$$

subject to

$$y(0) = 0, Dy(0) = 0 \text{ and } D^2y(0) = 0, \text{ where } D = \frac{d}{dx}.$$

- (b) Find the solution $y(x)$ in the explicit form for the following initial value problem:

$$\frac{dy}{dx} = \frac{x}{1+y}; \quad y(0) = 1.$$

- (c) Solve the following second-order linear differential equation:

$$8 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - y = 0; \quad y(-1) = 1, \frac{dy}{dx}|_{x=-1} = 0.$$

Show the details of your work.

[5 + 2 + 3 Marks]